Part IIA: Neoclassical Model of Labor Supply I

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Motivation

- Goal: Model individual labor supply decisions with a simple model to explain and help understand stylized facts about the labor market.
- Questions we will try to answer:
  - How many hours should an individual choose to work?
  - What factors motivate a person to enter the labor force in the first place?
  - How do individuals respond to tax breaks and other government policies in terms of their labor decisions?

Model Overview

- The Neoclassical Model of Labor Supply consists of two general pieces:
  1. Worker Preferences
     - What is an individual's goal when making decisions about how much to work?
     - Preferences over goods $\Rightarrow$ utility function
     - Individuals wish to maximize **utility**
  2. Constraints
     - What prevents people from never working and consuming an infinite amount?
     - Limited resources (e.g., time & income) constrain behavior

Choice Variables

- Individuals are free to choose both
  1. How much to consume, C
     - Opportunity cost: Leisure time
     - Measured in dollar units
  2. How much time to engage in leisure, L
     - Opportunity cost: Lost wages
- Trade-off: More leisure time $\Rightarrow$ Less work hours $\Rightarrow$ Less consumption

Worker Preferences

- Which is better: 100 hours of leisure and $400 of consumption or 90 hours of leisure and $600 of consumption per week?
- Depends on the preferences of a particular worker.
- Need a way to measure a worker's well-being from their chosen bundle of consumption and leisure ⇒ **utility function**, $U(C, L)$
- The utility function transforms consumption of goods and leisure into an index that measures "satisfaction" or "happiness"

Worker Preferences

Example

A worker's preferences are represented by the utility function $U(C, L) = C^{1/2}L^{1/2}$, where C is measured in dollars and L is measured in hours. How much utility does the worker receive from the bundles above?

- Workers **strictly prefer** bundles with higher levels of utility since more utility $\Rightarrow$ greater well-being
- If two bundles give a worker the same level of utility, we say the worker is **indifferent** between the bundles

Worker Preferences

- We want worker preferences to be rational or "well-behaved," so we assume the following:
  1. Completeness: Workers can always rank any two bundles as to their desirability
  2. Transitivity: Workers are consistent in their ranking of bundles
  3. Monotonicity:
     - A bundle with more of either consumption or leisure is always at least as good as a bundle with less of either good AND
     - A bundle with more of both consumption and leisure is always strictly preferred to a bundle with less of both goods

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Worker Preferences

- Preferences can be more easily visualized through indifference curves
- Indifference curves give the set of all bundles (C, L) that provide a particular utility level, $\overline{U}$
- Objective: Reach the highest indifference curve

Indifference Curves

Example

Consider "Cobb-Douglas" preferences: $U(C, L) = C^{1/2}L^{1/2}$.

a. *If a worker consumes \$200 worth of consumption goods, how many hours of leisure must he consume in order to be just as well off as if she consumes \$400 worth of consumption goods and 100 hours worth of leisure in a week?*
b. *Find the equation representing the indifference curve for the utility level* $\overline{U}$, *solved for* C.

Indifference Curves

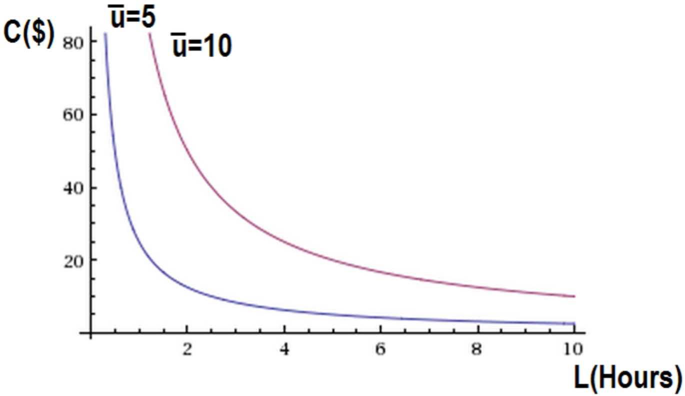

Figure: Sample Indifference Curves for $U = C^{1/2}L^{1/2}$

Indifference Curve Properties

1. Indifference curves are downward sloping
   - Implied by the strict monotonicity assumption
   - Upward sloping indifference curve would imply that a bundle with more C and L would yield the same level of utility as a bundle with less C and L
   - The only way to increase either consumption or leisure while holding utility constant is to take away some of the other good
2. Indifference curves further from the origin represent higher utility levels
   - Implied by monotonicity.
   - Bundles further from the origin contain more of either C and L (or both), so they will yield greater levels of utility than bundles close to the origin
3. Indifference curves are "thin"
4. **Indifference curves never cross**

Indifference Curve Properties

- Additional assumption: Preferences are convex, meaning that indifference curves are bowed towards the origin
- In words: workers prefer averages to extremes
- Does this make sense?

The Slope of an Indifference Curve

- As with most economic decisions, we assume workers think on the margin (e.g., "Should I take one more hour of leisure?")
- **Marginal Utility of Leisure (MU_L):** The increase in utility associated with an additional unit (hour) of leisure (holding C constant)
- **Marginal Utility of Consumption (MU_C):** The increase in utility associated with an additional unit (dollar) of consumption (holding L constant)

The Slope of an Indifference Curve

- Behavioral implications of convex preferences are best illustrated using the **marginal rate of substitution:**

$$MRS_{L,C} = \frac{MU_L}{MU_C} = \left| \frac{\Delta C}{\Delta L} \right|$$

- Graphically: $MRS_{L,C}$ is the (absolute) slope of a given indifference curve $C = f(L)$
- Verbally: $MRS_{L,C}$ is the maximum number of consumption dollars an individual is willing to sacrifice for an additional hour of leisure

The Slope of an Indifference Curve

- The convexity assumption implies a **diminishing marginal rate of substitution:**
  1. $\uparrow C \Rightarrow \uparrow MRS_{L,C}$
     - When consumption rises, workers are willing to sacrifice more consumption dollars for an additional leisure hours
  2. $\uparrow L \Rightarrow \downarrow MRS_{L,C}$
     - When leisure rises, workers are willing to sacrifice fewer consumption dollars for an additional hour of leisure

Readings

- Borjas 2.3

Labor Supply Constraints

- In the Neoclassical Model of Labor Supply, an individual chooses bundles of consumption and leisure
- Each worker is constrained by both (i) their income and (ii) time

The Consumption Constraint

- An individual can earn some income independently of how many hours they work (e.g., dividends, property income, etc.)
  - Referred to as "non-labor income"
  - Denoted V
- The individual can also decide to work some hours, h, and earn an hourly wage w
- A person's budget constraint is thus

$$C = wh + V,$$

  where C is "dollars of consumption"
- Anything missing in this simple constraint?

The Consumption Constraint

- For now, we will assume that the wage rate is constant for a particular individual
- In reality, the "marginal" wage rate generally depends on how many hours an individual has worked
  - Progressive taxes
  - Overtime pay

The Time Constraint

- Workers can allot their total time (T) between either leisure (L) or work (h)
- The time constraint is thus

$$T = h + L$$

The Budget Constraint

- We can rewrite the time constraint as $h = T - L$
- Plugging this into the consumption constraint allows us to write out the **budget constraint**:

$$C = w(T - L) + V = (wT + V) - wL$$

The Budget Constraint

- The slope of the budget constraint is $(-w)$
- If a worker chooses not to work ($h = 0$), then $T = L$ and the individual can consume up to V dollars. This point is called the **endowment point**
- If a worker chooses not to participate in leisure ($L = 0$), the worker can consume $wT + V$ dollars

The Budget Constraint

Example

Suppose there are 110 (non-sleeping) hours in a week available to split between work and leisure. A worker earns $10 per hour after taxes. Additionally, she also receives $320 worth of welfare benefits each week regardless of the number of hours she works. Graph her budget line.

The Budget Constraint

- Affordable bundles of consumption and leisure fall on or below the budget constraint
- The set of all affordable bundles is referred to as the **budget set**
- An individual's goal is to choose the best bundle within their budget set (i.e., choose the affordable bundle that maximizes utility)

The Budget Constraint

- What are “prices” in this context?
- Recall that the slope of a budget line is the (negative of the) ratio of the two commodities’ prices
- Here, the slope is $-w = -\frac{P_L}{P_C}$
  - We defined the price of consumption (P_C) as one dollar, so $P_C = 1$
  - P_L is the price of leisure. This represents the opportunity cost of an hour of leisure. Hence, $P_L = w$

The Budget Constraint

- "Exogenous" changes in the budget constraint are due to
  1. Changes in non-labor income (V)
  2. Changes in the wage rate (w)
- How do changes in each affect the budget line?

The Budget Constraint

Example

Tom earns \$15 per hour for up to 40 hours of work each week. He is paid \$30 per hour for every hour in excess of 40. Tom faces a 20% tax rate and pays \$4 per hour in child care expenses for each hour he works. Tom receives \$80 in child support payments each week. There are 110 (non-sleeping) hours in the week. Graph Tom's weekly budget line.

The Budget Constraint

- Policy dictates that the net wage, $w = w^G(1 - \tau)$ depends on the income tax rate τ
- Developed nations generally favor progressive tax structures: earnings of higher income individuals are taxed at an increasing rate.

If Taxable Income Is Between:	The Tax Due Is:
0 - $9,225	10% of taxable income
$9,226 - $37,450	$922.50 + 15% of the amount over $9,225
$37,451 - $90,750	$5,156.25 + 25% of the amount over $37,450
$90,751 - $189,300	$18,481.25 + 28% of the amount over $90,750
$189,301 - $411,500	$46,075.25 + 33% of the amount over $189,300
$411,501 - $413,200	$119,401.25 + 35% of the amount over $411,500
$413,201 +	$119,996.25 + 39.6% of the amount over $413,200

Figure: Single filer Tax Brackets, 2015

- Implications for budget constraint?

Readings

- Borjas 2.4

Motivation

- The whole reason we develop this model is to consider the impact of changes in incentives (i.e., policy)
- Today:
  - How does a worker choose the optimal number of work hours?
  - How do changes in non-labor income affect the hours-worked decision?
  - How do changes in the wage rate affect the hours-worked decision?

The Hours Decision

- Given an individual's wage rate and non-labor income, what is her optimal bundle of (C, L)?
- Individual's objective: Find the affordable bundle which yields the highest utility

The Hours Decision

- Graphically: Which bundle (C, L) within the budget set reaches the highest indifference curve?
- If a worker chooses to work, (i.e., $h > 0$), the optimal bundle will lie at the point where the indifference curve is **tangent** to the budget constraint
  - We will cover the participation decision next time

The Hours Decision

- Recall from Econ 310/410 that at an interior optimum, the *MRS* is equal to the price ratio of the two commodities
- In our model at the interior optimum, we have

$$MRS_{L,C} = \frac{MU_L}{MU_C} = w$$

- The price of an hour of leisure is forgone wages, $P_L = w$
- The price of consumption is defined as $P_C = 1$
- So, we have that $MRS =$ Price ratio at the interior optimum.

The Hours Decision

- What's the intuition behind the tangency condition?
- Rewrite the condition as

$$\frac{MU_L}{w} = \frac{MU_C}{\$1}$$

- Left hand side gives the number of utils received from spending an additional dollar on leisure (since each hour of leisure costs w)
- Right had side gives the number of utils received from spending an additional dollar on consumption

The Hours Decision

- If the two were not equal, a worker could rearrange their bundle so as to purchase more of the commodity that yields more utility for the last dollar
- How should a worker rearrange their bundle if
  a. $MRS > w$?
  b. $MRS < w$?

Example

Shelley's preferences for consumption and leisure can be expressed as $U(C, L) = (C - 100)(L - 40)$*. This implies* $MU_L = C - 100$ *and* $MU_C = L - 40$*. There are 110 available hours each week to split between work and leisure. Shelley earns $10 per hour after taxes and receives $320 worth of welfare benefits each week regardless of how much she works.*

a. *What is the equation for Shelley's budget line?*
b. *What is the equation for Shelley's* $MRS_{L,C}$*?*
c. *What is Shelley's optimal amount of consumption and leisure?*

Comparative Statics: A Change in Non-Labor Income

- The impact of the change in non-labor income (holding w constant) on the number of hours worked is called the **income effect**
- Two possible responses in hours worked due to a change in V:

  1. If leisure is *income normal*, $\uparrow V \Rightarrow \uparrow L^*, \downarrow h^*$
  2. If leisure is *income inferior*, $\uparrow V \Rightarrow \downarrow L^*, \uparrow h^*$

- From here forward, we will assume leisure is income-normal (consistent with empirical evidence)

Comparative Statics: A Change in Wages

- Two reasonable responses behind an increase in the wage rate:
    1. "An increase in wages increases my income, so I don't need to work as many hours in order to enjoy my desired quality of life"
    2. "An increase in wages makes leisure time more costly, so I will work more"
- Which one is right?

Comparative Statics: A Change in Wages

- The change in an individual's hours worked from a change in the wage rate can be decomposed into two factors:
  1. **Income Effect:** The change in an individual's leisure allocation resulting from the change in the individual's budget set, controlling for the substitution effect
  2. **Substitution Effect:** The change in an individual's leisure allocation resulting from the change in the relative price of leisure and consumption, w, controlling for the income effect

Comparative Statics: A Change in Wages

- Case 1: Substitution effect dominates the income effect
  - An increase in the wage rate leads to an increase in the number of hours worked
- Case 2: Income effect dominates the substitution effect
  - An increase in the wage rate leads to a decrease in the number of hours worked

Readings

- Borjas 2.5